

Contents

Preface

xiii

1 Risk in Perspective

1

1.1 Risk

1

1.1.1 Risk and Randomness

1

1.1.2 Financial Risk

2

1.1.3 Measurement and Management

3

1.2 A Brief History of Risk Management

5

1.2.1 From Babylon to Wall Street

5

1.2.2 The Road to Regulation

8

1.3 The New Regulatory Framework

10

1.3.1 Basel II

10

1.3.2 Solvency 2

13

1.4 Why Manage Financial Risk?

15

1.4.1 A Societal View

15

1.4.2 The Shareholder's View

16

1.4.3 Economic Capital

18

1.5 Quantitative Risk Management

19

1.5.1 The Nature of the Challenge

19

1.5.2 QRM for the Future

22

2 Basic Concepts in Risk Management

25

2.1 Risk Factors and Loss Distributions

25

2.1.1 General Definitions

25

2.1.2 Conditional and Unconditional Loss Distribution

28

2.1.3 Mapping of Risks: Some Examples

29

2.2 Risk Measurement

34

2.2.1 Approaches to Risk Measurement

34

2.2.2 Value-at-Risk

37

2.2.3 Further Comments on VaR

40

2.2.4 Other Risk Measures Based on Loss Distributions

43

2.3 Standard Methods for Market Risks

48

2.3.1 Variance–Covariance Method

48

2.3.2 Historical Simulation

50

2.3.3 Monte Carlo

52

2.3.4 Losses over Several Periods and Scaling

53

2.3.5 Backtesting

55

2.3.6 An Illustrative Example

55

3	Multivariate Models	61
3.1	Basics of Multivariate Modelling	61
3.1.1	Random Vectors and Their Distributions	62
3.1.2	Standard Estimators of Covariance and Correlation	64
3.1.3	The Multivariate Normal Distribution	66
3.1.4	Testing Normality and Multivariate Normality	68
3.2	Normal Mixture Distributions	73
3.2.1	Normal Variance Mixtures	73
3.2.2	Normal Mean-Variance Mixtures	77
3.2.3	Generalized Hyperbolic Distributions	78
3.2.4	Fitting Generalized Hyperbolic Distributions to Data	81
3.2.5	Empirical Examples	84
3.3	Spherical and Elliptical Distributions	89
3.3.1	Spherical Distributions	89
3.3.2	Elliptical Distributions	93
3.3.3	Properties of Elliptical Distributions	95
3.3.4	Estimating Dispersion and Correlation	96
3.3.5	Testing for Elliptical Symmetry	99
3.4	Dimension Reduction Techniques	103
3.4.1	Factor Models	103
3.4.2	Statistical Calibration Strategies	105
3.4.3	Regression Analysis of Factor Models	106
3.4.4	Principal Component Analysis	109
4	Financial Time Series	116
4.1	Empirical Analyses of Financial Time Series	117
4.1.1	Stylized Facts	117
4.1.2	Multivariate Stylized Facts	123
4.2	Fundamentals of Time Series Analysis	125
4.2.1	Basic Definitions	125
4.2.2	ARMA Processes	128
4.2.3	Analysis in the Time Domain	132
4.2.4	Statistical Analysis of Time Series	134
4.2.5	Prediction	136
4.3	GARCH Models for Changing Volatility	139
4.3.1	ARCH Processes	139
4.3.2	GARCH Processes	145
4.3.3	Simple Extensions of the GARCH Model	148
4.3.4	Fitting GARCH Models to Data	150
4.4	Volatility Models and Risk Estimation	158
4.4.1	Volatility Forecasting	158
4.4.2	Conditional Risk Measurement	160
4.4.3	Backtesting	162
4.5	Fundamentals of Multivariate Time Series	164
4.5.1	Basic Definitions	164
4.5.2	Analysis in the Time Domain	166
4.5.3	Multivariate ARMA Processes	168
4.6	Multivariate GARCH Processes	170
4.6.1	General Structure of Models	170
4.6.2	Models for Conditional Correlation	172
4.6.3	Models for Conditional Covariance	175

4.6.4	Fitting Multivariate GARCH Models	178
4.6.5	Dimension Reduction in MGARCH	179
4.6.6	MGARCH and Conditional Risk Measurement	182
5	Copulas and Dependence	184
5.1	Copulas	184
5.1.1	Basic Properties	185
5.1.2	Examples of Copulas	189
5.1.3	Meta Distributions	192
5.1.4	Simulation of Copulas and Meta Distributions	193
5.1.5	Further Properties of Copulas	195
5.1.6	Perfect Dependence	199
5.2	Dependence Measures	201
5.2.1	Linear Correlation	201
5.2.2	Rank Correlation	206
5.2.3	Coefficients of Tail Dependence	208
5.3	Normal Mixture Copulas	210
5.3.1*	Tail Dependence	210
5.3.2	Rank Correlations	215
5.3.3	Skewed Normal Mixture Copulas	217
5.3.4	Grouped Normal Mixture Copulas	218
5.4	Archimedean Copulas	220
5.4.1	Bivariate Archimedean Copulas	220
5.4.2	Multivariate Archimedean Copulas	222
5.4.3	Non-exchangeable Archimedean Copulas	224
5.5	Fitting Copulas to Data	228
5.5.1	Method-of-Moments using Rank Correlation	229
5.5.2	Forming a Pseudo-Sample from the Copula	232
5.5.3	Maximum Likelihood Estimation	234
6	Aggregate Risk	238
6.1	Coherent Measures of Risk	238
6.1.1	The Axioms of Coherence	238
6.1.2	Value-at-Risk	241
6.1.3	Coherent Risk Measures Based on Loss Distributions	243
6.1.4	Coherent Risk Measures as Generalized Scenarios	244
6.1.5	Mean-VaR Portfolio Optimization	246
6.2	Bounds for Aggregate Risks	248
6.2.1	The General Fréchet Problem	248
6.2.2	The Case of VaR	250
6.3	Capital Allocation	256
6.3.1	The Allocation Problem	256
6.3.2	The Euler Principle and Examples	257
6.3.3	Economic Justification of the Euler Principle	261
7	Extreme Value Theory	264
7.1	Maxima	264
7.1.1	Generalized Extreme Value Distribution	265
7.1.2	Maximum Domains of Attraction	267
7.1.3	Maxima of Strictly Stationary Time Series	270
7.1.4	The Block Maxima Method	271

7.2	Threshold Exceedances	275
7.2.1	Generalized Pareto Distribution	275
7.2.2	Modelling Excess Losses	278
7.2.3	Modelling Tails and Measures of Tail Risk	282
7.2.4	The Hill Method	286
7.2.5	Simulation Study of EVT Quantile Estimators	289
7.2.6	Conditional EVT for Financial Time Series	291
7.3	Tails of Specific Models	293
7.3.1	Domain of Attraction of Fréchet Distribution	293
7.3.2	Domain of Attraction of Gumbel Distribution	294
7.3.3	Mixture Models	295
7.4	Point Process Models	298
7.4.1	Threshold Exceedances for Strict White Noise	299
7.4.2	The POT Model	301
7.4.3	Self-Exciting Processes	306
7.4.4	A Self-Exciting POT Model	307
7.5	Multivariate Maxima	311
7.5.1	Multivariate Extreme Value Copulas	311
7.5.2	Copulas for Multivariate Minima	314
7.5.3	Copula Domains of Attraction	314
7.5.4	Modelling Multivariate Block Maxima	317
7.6	Multivariate Threshold Exceedances	319
7.6.1	Threshold Models Using EV Copulas	319
7.6.2	Fitting a Multivariate Tail Model	320
7.6.3	Threshold Copulas and Their Limits	322
8	Credit Risk Management	327
8.1	Introduction to Credit Risk Modelling	327
8.1.1	Credit Risk Models	327
8.1.2	The Nature of the Challenge	329
8.2	Structural Models of Default	331
8.2.1	The Merton Model	331
8.2.2	Pricing in Merton's Model	332
8.2.3	The KMV Model	336
8.2.4	Models Based on Credit Migration	338
8.2.5	Multivariate Firm-Value Models	342
8.3	Threshold Models	343
8.3.1	Notation for One-Period Portfolio Models	344
8.3.2	Threshold Models and Copulas	345
8.3.3	Industry Examples	347
8.3.4	Models Based on Alternative Copulas	348
8.3.5	Model Risk Issues	350
8.4	The Mixture Model Approach	352
8.4.1	One-Factor Bernoulli Mixture Models	353
8.4.2	CreditRisk+	356
8.4.3	Asymptotics for Large Portfolios	357
8.4.4	Threshold Models as Mixture Models	359
8.4.5	Model-Theoretic Aspects of Basel II	362
8.4.6	Model Risk Issues	364
8.5	Monte Carlo Methods	367
8.5.1	Basics of Importance Sampling	367
8.5.2	Application to Bernoulli-Mixture Models	370

8.6	Statistical Inference for Mixture Models	374
8.6.1	Motivation	374
8.6.2	Exchangeable Bernoulli-Mixture Models	375
8.6.3	Mixture Models as GLMMs	377
8.6.4	One-Factor Model with Rating Effect	381
9	Dynamic Credit Risk Models	385
9.1	Credit Derivatives	386
9.1.1	Overview	386
9.1.2	Single-Name Credit Derivatives	387
9.1.3	Portfolio Credit Derivatives	389
9.2	Mathematical Tools	392
9.2.1	Random Times and Hazard Rates	393
9.2.2	Modelling Additional Information	395
9.2.3	Doubly Stochastic Random Times	397
9.3	Financial and Actuarial Pricing of Credit Risk	400
9.3.1	Physical and Risk-Neutral Probability Measure	401
9.3.2	Risk-Neutral Pricing and Market Completeness	405
9.3.3	Martingale Modelling	408
9.3.4	The Actuarial Approach to Credit Risk Pricing	411
9.4	Pricing with Doubly Stochastic Default Times	414
9.4.1	Recovery Payments of Corporate Bonds	414
9.4.2	The Model	415
9.4.3	Pricing Formulas	416
9.4.4	Applications	418
9.5	Affine Models	421
9.5.1	Basic Results	422
9.5.2	The CIR Square-Root Diffusion	423
9.5.3	Extensions	425
9.6	Conditionally Independent Defaults	429
9.6.1	Reduced-Form Models for Portfolio Credit Risk	429
9.6.2	Conditionally Independent Default Times	431
9.6.3	Examples and Applications	435
9.7	Copula Models	440
9.7.1	Definition and General Properties	440
9.7.2	Factor Copula Models	444
9.8	Default Contagion in Reduced-Form Models	448
9.8.1	Default Contagion and Default Dependence	448
9.8.2	Information-Based Default Contagion	453
9.8.3	Interacting Intensities	456
10	Operational Risk and Insurance Analytics	463
10.1	Operational Risk in Perspective	463
10.1.1	A New Risk Class	463
10.1.2	The Elementary Approaches	465
10.1.3	Advanced Measurement Approaches	466
10.1.4	Operational Loss Data	468
10.2	Elements of Insurance Analytics	471
10.2.1	The Case for Actuarial Methodology	471
10.2.2	The Total Loss Amount	472
10.2.3	Approximations and Panjer Recursion	476
10.2.4	Poisson Mixtures	482

10.2.5	Tails of Aggregate Loss Distributions	484
10.2.6	The Homogeneous Poisson Process	484
10.2.7	Processes Related to the Poisson Process	487
Appendix		494
A.1	Miscellaneous Definitions and Results	494
A.1.1	Type of Distribution	494
A.1.2	Generalized Inverses and Quantiles	494
A.1.3	Karamata's Theorem	495
A.2	Probability Distributions	496
A.2.1	Beta	496
A.2.2	Exponential	496
A.2.3	F	496
A.2.4	Gamma	496
A.2.5	Generalized Inverse Gaussian	497
A.2.6	Inverse Gamma	497
A.2.7	Negative Binomial	498
A.2.8	Pareto	498
A.2.9	Stable	498
A.3	Likelihood Inference	499
A.3.1	Maximum Likelihood Estimators	499
A.3.2	Asymptotic Results: Scalar Parameter	499
A.3.3	Asymptotic Results: Vector of Parameters	500
A.3.4	Wald Test and Confidence Intervals	501
A.3.5	Likelihood Ratio Test and Confidence Intervals	501
A.3.6	Akaike Information Criterion	502
References		503
Index		529